

Axial Yield Strengths and Two Successive Phase Transition Stresses for Crystalline Silicon*†

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Shock wave experiments with inclined prisms have been used to determine the compression for shocks propagated along three axes of single-crystal Si. At stresses less than about 200 kbar, the shocks along the $\langle 110 \rangle$ and $\langle 111 \rangle$ axes exhibited four wave structures. The first three waves were taken to be related to the Hugoniot Elastic Limit and to two succeeding polymorphic phase transitions. Scatter in the magnitude of the HEL along the $\langle 100 \rangle$ axis may indicate mixing between the elastic wave and the first phase transition wave.

INTRODUCTION

In 1962, Minomura and Drickamer¹ reported finding a substantial change in the electrical resistivity of crystalline Si resulting from static compression at about 200 kbar. This change, of five or six orders-of-magnitude, was attributed to a solid-solid phase transition from the semiconductor to the metallic state. When large shear or uniaxial stresses were present, the authors found the change to occur at 135–150 kbar.

The following year, Jamieson² reported x-ray diffraction work showing the transformation of Si from the diamond structure to the white tin structure, with a volume reduction of 22% at pressures less than ~ 160 kbar. He also noted the presence of two extra diffraction lines which did not fit the white tin structure. These lines persisted during pressure reduction, indicating the presence of an additional phase. Wentorf and Kasper³ reported a reduction in the resistivity of high-purity Si by a factor of 10^2 or 10^3 at static pressure of only 110–120 kbar, followed by another reduction of 10^3 at 150–160 kbar. They found, after relaxation, a complicated bcc crystal structure with 16 atoms per unit cell and a density of 2.55 g/cm^3 instead of the normal 2.33 g/cm^3 .

McQueen⁴ cites dynamic experiments by Wackerle, who used the inclined mirror method to observe Hugoniot Elastic Limits (HEL) and phase transition pressures in Si. A broad, slowly rising plastic wave prevented Wackerle from resolving the transition pressure. In 1965, Al'tshuler⁵ reported Pavlovskii's findings of a 35-kbar HEL and a 100-kbar phase transition pressure. Pavlovskii⁶ later reported 40 kbar, $0.413 \text{ cm}^3/\text{g}$, and 112 kbar, $0.385 \text{ cm}^3/\text{g}$, as the (P, V) points, respectively, for the HEL and phase transition pressures along the $\langle 111 \rangle$ axis of Si.

The findings are somewhat contradictory. It was this that prompted the present investigations.

PROCEDURE

In materials exhibiting a dynamic yield point or Hugoniot Elastic Limit, a shock wave traversing a sample of the material may break up into two suc-

cessive waves. The first wave brings the material up to the plastic yield point, propagates roughly at longitudinal sound speed, and is commonly called the elastic wave. Plastic yielding occurs in the second, or plastic, wavefront. In the case of a polymorphic phase transition, this second wave, commonly called the transition wave, brings the material up to the point of

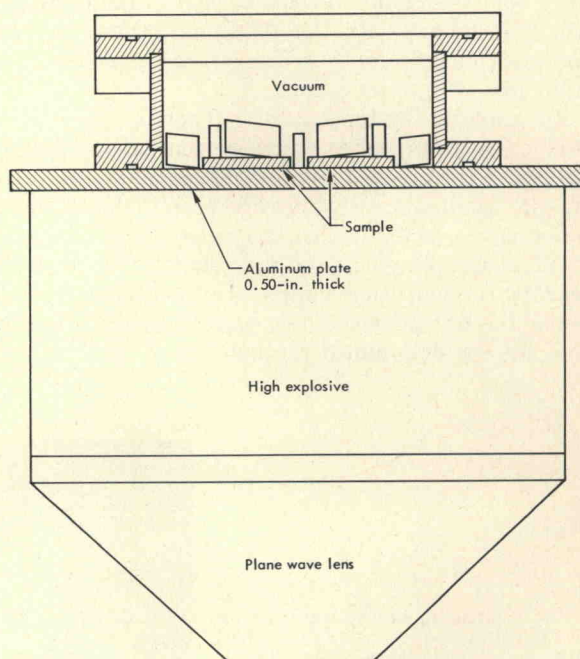


FIG. 1. Inclined prism geometry.

transformation. The actual transformation takes place in a third wave. Additional transformations may take place in succeeding waves. Observation of the amplitude of the second (transition) wave allows the experimental determination of the phase transition stress.

We used the inclined prism technique⁷ to observe multiple wave structures in shocks propagated along the $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ axes of extremely pure single-crystal Si (impurities present only in parts per

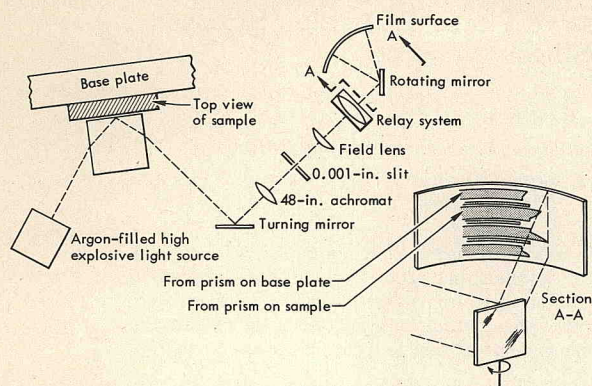


FIG. 2. Schematic drawing of inclined prism optical system.

billion). This technique provides a reliable means of making continuous measurement of the free surface velocities associated with the emerging elastic, transition, and final waves. The method is based on the principle of extinction of total internal reflection within a prism. The prism contacts the sample surface at only a few points, and any movement of the sample surface into full contact with the reflecting surface changes the index of refraction of the gap medium, resulting in the loss of total reflection.

The explosive systems listed in Table I were used to produce final pressures ranging from about 110 to 300 kbar. The inclined part of the geometry, shown in Fig. 1, simultaneously provided continuous direct measurement of the free surface velocities of two areas of the base plate and of two samples. The shock velocity through each sample was measured through use of the flat prisms. Thus, particle velocity for the final state is determined through use of both the free

TABLE I. Shock-generating systems.^a

System	High explosive	Attenuator material	Component thickness (mm)
A	Pressed TNT	Brass	12.7
		Lucite	25.4
		Al	12.7
B	PBX 9205 (RDX)	Brass	12.7
		Lucite	19
		Al	12.7
C	Pressed TNT	Brass	12.7
		Al	12.7
D	Pressed TNT	Al	12.7
E	PBX 9205 (RDX)	Brass	25.4
		Al	12.7
F	PBX 9404 (HMX)	Brass	12.7
		Al	12.7
G	PBX 9205 (RDX)	Al	12.7
H	PBX 9404 (HMX)	Al	12.7
I	PBX 9404 (HMX)	Air	3.2
		Monel	3.2
		Air	25.4
		Al	4.8

^a All H.E. systems consisted of a 30.5-cm-diam plane-wave lens plus a 30.5-cm-diam right cylinder of high explosive 15.2-cm long. All attenuator components and base plates were flat and parallel within 5 μ .

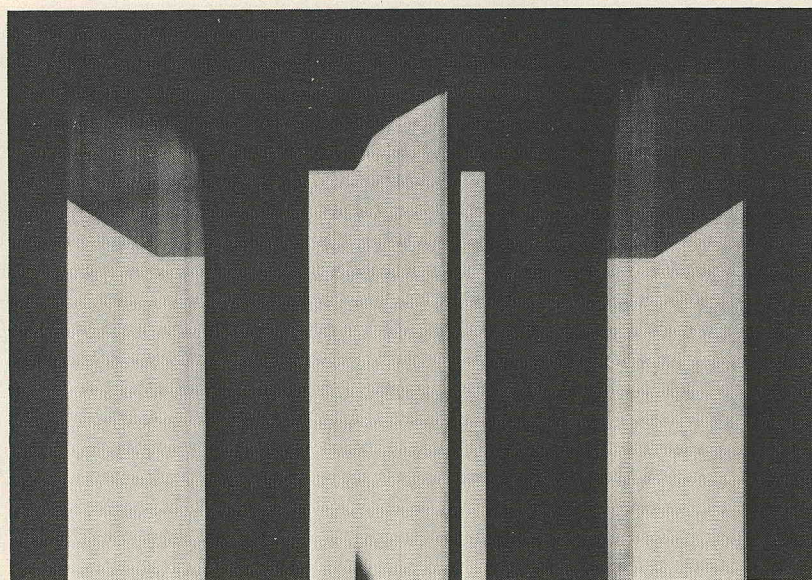


FIG. 3. Streaking camera record for 110 Si (3° prisms). Total time from shock breakout at the base plate (i.e., the first flat trace) to cutoff at the end of the record was 2.2 μ sec, time progressing from bottom to top. The first slope on the center trace is related to the HEL, the second to the 103-kbar transition, and the third to the 128-kbar transition.

TABLE II. Axial yield strengths and phase transition stresses (initial volume = 0.429 cm³/g).

Axis	Hugoniot elastic limit (kbar)	Volume (cm ³ /g)	1st Transition		2nd Transition	
			Stress (kbar)	Volume (cm ³ /g)	Stress (kbar)	Volume (cm ³ /g)
$\langle 100 \rangle$	92 ± 10	0.406	None observed		140 ± 4	0.385
$\langle 110 \rangle$	50 ± 5	0.419	103 ± 7	0.398	128 ± 5	0.385
$\langle 111 \rangle$	54 ± 3	0.418	101 ± 3	0.400	137 ± 5	0.383

surface approximation and impedance matching technique. The angles of inclination were 5° for the base plate prisms and either 3° or 5° for the prisms on the samples.

In this optical system the prisms are intensely illuminated from a high-explosive-driven argon light source and the reflected image is viewed with a rotating-mirror streaking camera (see Fig. 2). The initial shock causes the free surface to move across the wedge-shaped gap at velocity U_{fs1} , where it progressively contacts the prism surface along the inclined length, changing the index of refraction of the second medium. Total reflection along the prism then is progressively destroyed by the moving contact so that the light is progressively cut off. The velocity of the light-dark discontinuity is proportional to the material free-surface velocity and, when viewed through a small slit, is recorded as a slope on the streaking camera record. A subsequent wave moving with velocity $U_{s2} + U_{p1}$ overtakes the boundary moving at U_{fs1} , establishing a different, faster contact rate. This new rate results in a different (smaller) slope related to U_{fs2} , the velocity at which the surface now moves. A similar sequence takes place for any succeeding waves.

Figure 3 is a typical resulting photographic record.

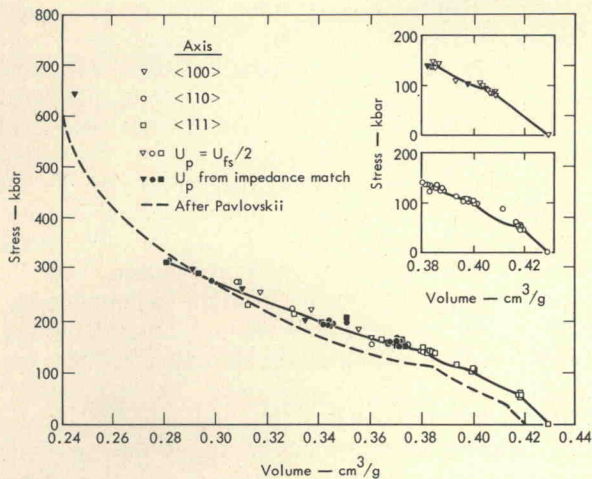


FIG. 4. Hugoniot plots for three orientations of crystalline Si.

This $\langle 111 \rangle$ sample trace shows cutoff angles for the HEL and two successive phase transitions. The cutoff angle for the final (fourth) wave in which the second phase transition takes place is not shown because this wave arrived after the observation period allowed by the 3° gap.

Detailed analysis of the reflections and interactions behind the front of a four-wave shock structure is beyond the scope of this paper. Hence, the data reduction procedure, similar to that used for inclined mirrors,⁸ uses apparent or average values for the shock velocities. The film record is related to the experiment through use of

$$U_{sn} = \{H + \sum_1^n U_{fs(n-1)}[t_n - t_{(n-1)}]\} / (t_n - t_0),$$

$$U_{fsn} = W \tan \alpha / M \tan \gamma_n,$$

$$U_{pn} = U_{fsn} / 2.$$

Here U_s is shock velocity, H is sample thickness, and $t = dx/W$ where x is either the flat prism cutoff position on the film record or the intersection point of the extrapolation for each of the successive free surface velocity slopes. The free surface velocity measurement depends on W , the camera streaking rate; M , the magnification factor relating the film record to actual experimental dimensions; α , the angle of prism inclination; and γ , the angle between the free surface trace and the normal to the camera sweep. Small corrections⁸ are applied to account for deviations from ideal geometry.

Application of the conservation of mass and momentum across the shock front leads to the Hugoniot equations which, for multiple wave structures, may be expressed in terms of pressure and volume as

$$P_n = V_{(n-1)}^{-1}(U_{sn} - U_{p(n-1)})(U_{pn} - U_{p(n-1)}) + P_{(n-1)},$$

$$V_n = V_{(n-1)}(U_{sn} - U_{pn}) / (U_{sn} - U_{p(n-1)}).$$

RESULTS

Our results are summarized in Table II; the complete data are presented in Tables III, IV, and V. In Fig. 4, the data are compared with results of Pavlovskii.

TABLE III. Summary

H.E. system	Al base plate pressure (kbar)	Sound speed at 1 atm (mm/ μ sec)		Elastic regime			
		C_L	C_S	Shock velocity, U_{s1} (mm/ μ sec)	Free sur- face veloc- ity, U_{fs1} (mm/ μ sec)	Stress, σ_1 (kbar)	Volume, V_1 (cm ³ /g)
A	99	8.30	5.79	8.30	0.865	83	0.4069
B	138	8.46	5.84	8.67	0.930	94	0.4062
C	162	8.38	5.89	8.67	0.815	82	0.4090
D	217	8.28	5.89	8.99	0.854	89	0.4088
E	223	8.51	5.89	8.64	0.857	86	0.4079
F	221	8.46	5.84	8.62	0.938	94	0.4058
G	309	8.51	5.91	8.55	1.007	100	0.4039
H	363	8.38	5.89	8.64	1.063	107	0.4028
I	787						

^a All samples were 6.4 mm thick; initial density was 2.330 g/cm³.

^b Data on this line and on all similar lines below were obtained from $U_p = U_{fs}/2$.

^c Data on this line and on all similar lines below were obtained from impedance matching.

DISCUSSION

Four discrete straight-line slopes were found on film records of experiments in which the shocks were propagated along the $\langle 110 \rangle$ and $\langle 111 \rangle$ axes. In each case the succeeding slope was of smaller value, thus indicating steps of increasing free-surface velocity. Close agreement between U_{s1} and C_L , the ultrasonically determined longitudinal sound speed at 1 atm,⁹ indicates that the first wave was elastic.

The possibility that the second wave also might be elastic was ruled out on the following grounds. The $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ directions in a cubic material all are directions in which the shear and longitudinal elastic waves are pure modes, though there are two shear waves in the $\langle 110 \rangle$ direction.¹⁰ Thus, there should be only one longitudinal elastic wave of concern in these experiments. Laue x-ray photographs showed the sample crystal structure to be within 1° of the desired orientation, so there should be no mixed modes caused by misorientation. The geometry of the experiments was designed such that the compression was entirely uniaxial, lateral boundaries being far from the region of measurement. Thus, from symmetry, there is no way for shear waves to be launched into the samples.

The possibility that a reverberation from the high

explosive and attenuator plate system overtaking the main shock would result in the appearance of a spurious multiple-wave structure was ruled out by the construction of time vs distance plots of the wave propagation involved.

The tabulated results show that the nominal values of the shock and particle velocities of the first, second, and third waves are more or less independent of the type and strength of the shock system. This provides strong evidence that these waves are related to phenomena characteristic of the material rather than to external loading conditions, and we can conclude that the second and third waves are related to two successive shock-induced phase transitions.

Four-wave structures were observed in experiments with the usual 6-mm samples, but because of the short time intervals between waves, details observable on the original negatives became fuzzy and unconvincing in reproductions. Better wave resolution was obtained by using 3° prism angles with 9.5-mm samples. However, in this case the available observation time was not long enough to record the entire event. For experiments where the entire event was not monitored, we noted a large disparity when the final pressures obtained from impedance matching were compared with those from the free surface approximation, as would be expected.

of data, Si $\langle 100 \rangle$.^a

Second wave regime				Third wave regime			
Shock velocity, U_{s2} (mm/ μ sec)	Particle velocity, U_{p2} (mm/ μ sec)	Stress σ_2 (kbar)	Volume, V_2 (cm ³ /g)	Shock velocity, U_{s3} (mm/ μ sec)	Particle velocity, U_{p3} (mm/ μ sec)	Final stress, σ_3 (kbar)	Volume, V_3 (cm ³ /g)
6.07	0.623 ^b	109	0.3931				
	0.56 ^e	101	0.3977				
6.32	0.732	132	0.3877				
	0.775	138	0.3847				
6.32	0.788	137	0.3826				
	0.915 ^d	157					
6.80	0.769	142	0.3869				
	1.15 ^d	201					
6.46	0.781	138	0.3840	4.84	1.03 ^b	164	0.3606
					1.30 ^e	196	0.3351
6.29	0.763	136	0.3853	5.38	1.12	179	0.3555
					e		
6.67	0.794	145	0.3849	5.55	1.38	217	0.3371
					1.71	256	0.3104
Not observed				5.75	1.64	250	0.3173
					1.98	294	0.2911
				8.0	3.43 ^f	640	0.2449

^d Incorrect data; the disparity indicates that a third wave existed, but the observation time was not long enough to record it.

^e No impedance match because of partial experimental failure.

^f From impedance matching of flash gap data.

Figure 3 depicts this; the slopes for $\langle 110 \rangle$ silicon are related to the HEL and two succeeding phase transitions. The final wave is not visible on the record.

The point of primary interest in Fig. 3 is the second slope (i.e., that related to the first phase transition), since the existence of two successive phase transitions is the unexpected result of this work. The record in this figure is the clearest evidence of the 102-kbar transition that could be reproduced without serious loss of detail.

Shocks propagated along the $\langle 100 \rangle$ axis had only three resolvable components; the first was the elastic wave and the second a phase transition wave occurring at nearly the same (P , V) point as the second phase-transition wave for the $\langle 110 \rangle$ and $\langle 111 \rangle$ axes. The record was very fuzzy in the region of the film record where one might expect a discontinuity related to the first phase transition, suggesting the possibility of two competing waves of nearly equal shock velocities.

In the $\langle 100 \rangle$ direction, the HEL appears to increase from about 82 to 107 kbar as driving pressure is varied from 100 to 360 kbar. This scatter may well be related to mixing between the HEL and an unresolved phase transition which should correspond to the 102-kbar transition found for the $\langle 110 \rangle$ and $\langle 111 \rangle$ axes. If this is so, the data can be divided such that the HEL occurs at about 85 kbar with a phase transition at

about 100 kbar. Experiments with thicker samples, allowing greater transit time differences, might permit resolution of the two waves.

For the $\langle 110 \rangle$ orientation, no consistent variation of shock amplitude with sample thickness or shock propagation distance was noted.

The discovery of two successive phase transitions in Si has the effect of qualitatively correlating the high-pressure x-ray, hydrostatic, and dynamic data. In particular, the persistence of two unidentified lines not related to the white tin structure, noted by Jamieson,² the two-step change in resistance found by Wentorf and Kasper,³ and the post-experimental volume of the bcc structure (0.392 cm³/g) fit very nicely. The latter volume should be compared with the state achieved just before onset of the second phase transition, namely, 0.383–0.385 cm³/g.

Data reported by Pavlovskii⁶ are listed in Table VI. He did not report what we term the "first phase transition," though his oscillograms clearly show two discontinuities in the region of interest. Further, he concludes from scaling and the form of the (P , V) curve that the final phase at pressures above Jamieson's has a density greater than that of Si in the white tin structure, thus implying an additional phase transition. Data from the present work support this conclusion.

TABLE IV. Summary

H.E. system	Al base plate pressure (kbar)	Sound speed at 1 atm (mm/ μ sec)		Elastic regime				Second wave regime			
				Shock velocity, U_{s1} (mm/ μ sec)	Free surface velocity, U_{fs1} (mm/ μ sec)	Stress, σ_1 (kbar)	Volume, V_1 (cm ³ /g)	Shock velocity, U_{s2} (mm/ μ sec)	Particle velocity, U_{p2} (mm/ μ sec)	Stress, σ_2 (kbar)	Volume, V_2 (cm ³ /g)
		C_L	C_S								
C	166	9.17	5.64 4.70	9.58	0.444	49	0.4192	7.07	0.576	107	0.3975
C	156	9.17	5.76 4.70	9.56	0.552	61	0.4168	f			
C	168	9.09	5.81 4.67	9.37 ^a	0.440	48	0.4191				
C	160	9.10	5.83 4.66	9.54 ^a	0.453	50	0.4190	7.01	0.571	106	0.3978
C	169			9.27 ^b	0.475	51	0.4181	6.74	0.576	103	0.3964
C	167			9.45 ^b	0.441	48	0.4193	6.87	0.516	95	0.4007
D	215			9.63	0.504	56	0.4180	7.08	0.534	102	0.4006
D	222	9.09	5.84 4.67	9.37 ^a	0.434	47	0.4193				
D	226			9.41 ^a	0.452	49	0.4189	6.82	0.575	105	0.3967
D	230			9.39 ^b	0.455	50	0.4187	6.85	0.545	100	0.3986
D	223			9.37 ^b	0.500	55	0.4177	6.83	0.623	113	0.3940
D	238			8.92 ^b	0.436	45	0.4187				
F	221	9.14	5.99 4.65	9.49	0.408	45	0.4199	7.09	0.515	96	0.4009
G	338	9.14	5.84	9.10	0.444	47	0.4187	f			
H	392	9.42	5.74 4.47	9.56 ^d					0.388 ^d	86	0.4117

^a All samples were 6.4 mm thick except as noted; initial density was 2.330 g/cm³.

^b Data on this line and on all similar lines below were obtained from $U_p = U_{fs}/2$.

^c Data on this line and on all similar lines below were obtained from impedance matching.

^d Mixing of waves.

The volume points which Pavlovskii reports for the HEL and second phase transition agree very well with ours, but the pressures do not. The differences are caused primarily by disparity in the measured values of U_p . We also note a disagreement in the initial volumes used (see Fig. 4). The handbook density for crystalline Si is 2.332 g/cm³. Our 2.330-g/cm³ value measured by immersion of each of our samples compares favorably with the handbook value. Pavlovskii lists $\langle 111 \rangle$ Si with $V_0 = 0.426$ cm³/g, but the figures in the report indicate a V_0 of 0.42. If the (P, V) curve reported by the Soviet workers were translated to the proper initial volume, the apparent disagreement would be far less.

Furthermore, it may be noted in Table VI that the value reported for $D_1 (= U_{s1})$ differs from ours by about 12%. The longitudinal sound speed for $\langle 111 \rangle$ Si in Table V is 9.37 mm/ μ sec. This compares favorably with our value for U_{s1} , namely, 9.61 mm/ μ sec. Pavlovskii's value $D_1 = 8.5$ km/sec for $\langle 111 \rangle$ Si appears more consistent with the actual C_L for the $\langle 100 \rangle$ orientation.

While the measured values of U_{s1} are reasonably close to the values of the longitudinal sound velocity C_L measured on the samples, closer inspection reveals that the elastic precursor moves at $\sim 2\%$ – 3% faster than sonic velocity. Table VII gives the average measured longitudinal sound velocities,⁹ average veloci-

of data, Si $\langle 110 \rangle$.^a

Third wave regime				Fourth wave regime			
Shock velocity, U_{s3} (mm/ μ sec)	Particle velocity, U_{p3} (mm/ μ sec)	Stress, σ_3 (kbar)	Volume, V_3 (cm ³ /g)	Shock velocity, U_{s4} (mm/ μ sec)	Particle velocity, U_{p4} (mm/ μ sec)	Final stress, σ_4 (kbar)	Volume, V_4 (cm ³ /g)
6.28	0.750	128 ^b	0.3840				
	0.945	157 ^{e,e}	0.3705				
6.48	0.758	133	0.3844	4.83	1.45 ^{b,e}	206	0.3191
					1.90 ^e	148	0.3710
6.65	0.730	126	0.3859	5.62	0.804	135	0.3801
					0.963	156	0.3675
6.27	0.705	125	0.3883				
	0.955 ⁱ	160	0.3709				
6.10	0.746	126	0.3841				
	0.972 ⁱ	157	0.3679				
6.36	0.70	122	0.3882				
	0.945 ⁱ	158	0.3714				
6.45	0.745	133	0.3862	5.28	0.879	149	0.3748
					1.220	190	0.3458
6.70	0.687	120	0.3889	5.78	0.801	136	0.3798
					1.27 ⁱ	194	0.3441
6.09	0.779	133	0.3820				
6.21	0.752	129	0.3840				
	1.29 ⁱ	205	0.3464				
6.12	0.806	138	0.3808				
	1.25 ⁱ	199	0.3486				
6.36	0.735	121	0.3834	5.08	0.951	145	0.3606
					1.40 ⁱ	196	0.3247
6.33	0.770	133	0.3833	5.32	1.26	191	0.3420
					1.26	192	0.3416
6.60	0.741	126	0.3846	5.74	1.45	220	0.3298
					1.80	272	0.2985
				5.86	1.76	269	0.3082
					2.11	315	0.2820

^e Less certain data; taken at end of record or from a slightly rounded trace.^f Transition overdriven (discernible but too fuzzy to measure).^g Sample thickness was 9.5 mm.^h Sample thickness was 12.7 mm.ⁱ Disparity indicates that the observation time allowed by a 3° prism was not long enough to record a fully developed fourth wave.

ties of the elastic precursors, and a longitudinal sound velocity calculated from second-order elastic constants in the literature.¹¹ This systematic difference probably results from the elastic precursor not being a weak wave; i.e., the longitudinal elastic compressibility cannot be assumed to be linear.

Table VII also gives the elastic strain measured in the precursor wave and longitudinal wave velocities calculated from second- and third-order elastic constants in the literature.^{11,12} Holt^{10,12} gives formulae for calculating the directional dependence of the longitudinal modulus associated with the propagation of a weak longitudinal disturbance on top of a strong

longitudinal stress wave. The modulus associated with a longitudinal elastic shock wave, such as an elastic precursor, then becomes the average result of applying Holt's formulae, including third-order elastic constants and the analogous second-order formulae. This procedure is equivalent to working with the third-order stress-strain tensor relations but is considerably easier, since most of the work was done by Holt.

For the $\langle 110 \rangle$ and $\langle 111 \rangle$ orientations, the agreement is embarrassingly good when the measured precursor velocities are compared with predictions of the third-order theory. Clearly, fourth-order elasticity is negligible. This is quite unlike the situation in quartz,

TABLE V. Summary

H.E. system	Al base plate pressure (kbar)	Sound speed at 1 atm (mm/ μ sec)		Elastic regime				Second wave regime			
				Shock velocity, U_{s1}	Free surface velocity, U_{fs1}	Stress, σ_1	Volume, V_1	Shock velocity, U_{s2}	Particle velocity, U_{p2}	Stress, σ_2	Volume, V_2
		C_L	C_S	(mm/ μ sec)	(mm/ μ sec)	(kbar)	(cm ³ /g)	(mm/ μ sec)	(mm/ μ sec)	(kbar)	(cm ³ /g)
C	166	9.37	5.08	9.72	0.491	56	0.4183	7.08	0.543	104	0.4001
C	156	9.37	5.03	9.65	0.488	55	0.4183	e			
D	212	9.35	5.16	9.72	0.474	54	0.4187	e			
D	215	9.37	5.10	9.54	0.484	50	0.4183	6.97	0.538	97	0.3998
F	221	9.40	5.10	9.68	0.477	54	0.4186	e			
G	338	9.37	5.10	9.38	0.492	54	0.4179	6.47 ^d	0.623	110	0.3926
H	392	9.35	5.13	9.61	0.482	54	0.4184	7.10	0.537	102	0.4003

^a All samples were 6.4 mm thick; initial density was 2.330 g/cm³.^b Data on this line and on all similar lines below were obtained from $U_p = U_{fs}/2$.^c Data on this line and on all similar lines below were obtained from impedance matching.

TABLE VI. Data from Pavlovskii (Ref. 6).

	Elastic-Plastic state					Phase transition			
	D_1 (km/sec)	U_1 (km/sec)	P_1 (kbar)	V_1 (cm ³ /g)	V_0 (cm ³ /g)	D_2 (km/sec)	U_2 (km/sec)	P_2 (kbar)	V_2 (cm ³ /g)
$\langle 111 \rangle$ Si	8.5	0.20	40	0.413	0.426	6.70	0.66	112	0.384

TABLE VII. Longitudinal wave velocities (mm/ μ sec).

Measurement/Calculation	Velocity (mm/ μ sec)			Precision
	$\langle 100 \rangle$	$\langle 110 \rangle$	$\langle 111 \rangle$	
Average of ultrasonic measurements on samples used in this work	8.41	9.21	9.37	$\pm 1\%$
Calculated from second-order elastic constants	8.43	9.13	9.35	
Elastic precursor U_{s1}	8.64	9.49	9.61	$\pm 1\%$
Strain in elastic precursor	-0.053	-0.025	-0.025	$\pm 10\%$
Calculated from second- and third-order elastic constants	8.74	9.49	9.65	$\pm 1\%$

of data, Si $\langle 111 \rangle$.^a

Third wave regime				Fourth wave regime			
Shock velocity, U_{s3} (mm/ μ sec)	Particle velocity, U_{p3} (mm/ μ sec)	Stress, σ_3 (kbar)	Volume, V_3 (cm ³ /g)	Shock velocity, U_{s4} (mm/ μ sec)	Particle velocity, U_{p4} (mm/ μ sec)	Final stress, σ_4 (kbar)	Volume, V_4 (cm ³ /g)
6.40	0.777	138 ^b	0.3826				
	0.930	160 ^c	0.3722				
6.65	0.769	135	0.3840	5.02	0.958 ^b	156	0.3669
					0.895 ^c	148	0.3726
6.67	0.778	137	0.3825	5.11	0.991	159	0.3641
					1.15	201	0.3505
6.38	0.794	134	0.3809	5.20	0.918	148	0.3702
					1.22	188	0.3436
6.71	0.748	133	0.3856	5.25	1.39	208	0.3306
					1.26	192	0.3413
				5.29	1.57	224	0.3125
					1.80	286	0.2932
6.47	0.836	146	0.3801	5.83	1.78	269	0.3084
					2.14	309	0.2808

^d Mixing of waves.^e Transition overdriven (discernible but too fuzzy to measure).

where Fowles⁸ observed a precursor velocity significantly faster than predicted by the third-order theory. He found it necessary to invoke fourth-order elasticity to account for the results, and his experimental data were used to determine fourth-order constants.

For the $\langle 100 \rangle$ orientation, the measured velocity is about 1% slower than predicted by theory. This is completely consistent with the observed wave being a mixture of an elastic precursor and a transition wave. In this event, the value for elastic strain would be smaller than that used here for the third-order elasticity prediction; reducing this value would lower the predicted velocity by just the required amount.

Table VIII gives average shear velocities for the samples used in this work.⁹ The anisotropy exhibited by the variation of the HEL may be roughly correlated with the shear moduli for the different axes.

TABLE VIII. Shear wave velocities (mm/ μ sec).

Measurement/ Calculation	Velocity (mm/ μ sec)			Precision
	$\langle 100 \rangle$	$\langle 110 \rangle$	$\langle 111 \rangle$	
Average of ultrasonic measurements	5.87	5.79, 4.63	5.10	$\pm 1\%$
Calculated from elastic constants	5.84	5.84, 4.67	5.09	

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